**JAMES ROSADO**

**ENGINEERING ANALYSIS**

**SUMMER 2015**

**HOMEWORK #3 Problem 5**

Solve the wave equation: for and . For a vibrating rectangular membrane subject to the following boundary and initial conditions:

B.C.: and

I.C.: and

We will utilize separation of variables and let we get the new differential equation when we substitute,

Or for short hand,

Factor and then divide we obtain the following,

We have obtained two new simpler differential equations,

We will begin with the second equation, factor and move terms to either side of the equality and we get,

Then divide to get,

We yet again obtain two more differential equations,

The second equation has characteristic equation: . The solutions will be of the form,

The boundary conditions imply: . By the first condition we get, . Hence , if we consider the second condition we get,

This means that and the corresponding solutions will be,

Let us now find . We established,

The solutions to the characteristic equation are of the form, . Therefore, the solution to ϕ will be of the form: . With the first set of boundary conditions we get: . For we arrive at the same result as before A = 0 and . With the second boundary conditions we have, . Therefore,

But we have determine μ,

The solution for ϕ (x) we be,

Let us finally analyze the temporal part of our solution: . The solutions to its characteristic equation are . Remember we have found earlier. The solutions to the temporal equation will be of the form,

If we combine all of our solutions a particular solution to the original 2-d wave equation is of the form,

Let us look at m = 1,2,3:

Each one of these solutions can have linear combinations of sines and cosines and by the superposition principle we get,

But then the sum of each of these solutions is also a solution: is a solution. Hence we arrive at the double summation,

Where,

We will use the initial condition, to find the coefficients.

We will call the inner summation on ‘n’ :

Thus,

But recall that,

Hence,

Upon rearranging and simplifying we get,

Therefore,

We will find the ‘B’ coefficients in a similar way except we will find the derivative of our solution,

If we let t = 0, we get,

Or simply,

We will follow the same procedure as before and let the inner sum be ,

Therefore,

So ; and,

Again we can find the ‘B’ coefficients using integrals,

Thus, This means our solution depends entirely on the ‘A’ coefficients,

The complete solution is,